antares

White paper

[Risk aggregation using Monte Carlo simulation 3.0

[Introduction

This document describes the functionality of the simulation module 'Risk aggregation using Monte Carlo simulation'.

antares RiMIS® is a standard software for opportunity and risk management. Elementary components of this standard software are the management of measures, the reporting system, the mapping of the ICS process, as well as the processing of compliance risks and risk identification by means of questionnaires.

The standard software is primarily designed for the industrial, service and trade sectors, for corporate groups, stock corporations and larger medium-sized companies.

Aggregation using Monte Carlo been available since version 3.0, which was released in September 2010. This module is not part of the standard license for antares RiMIS®, so it may have to be licensed separately. To understand this document, some basic knowledge of stochastics and statistics is required.

In the following, highly simplified examples are presented for illustrative purposes, which, however, have limited realism as a result, e.g., the number of run-throughs or histogram bars is greatly reduced.

[1. Purpose of the extension module

The module for aggregation by means of Monte Carlo simulation is used to implement this scientifically recognized method of aggregating risks from different subject areas or organizational units. Due to their random occurrence and their random nature, risks must not simply be added together, as this would result in a loss of any diversification effect. By means of a very large number of random simulation runs, an overall risk distribution is formed in each case, taking into account the probability distributions for occurrence and expression of the individual risks, this risk distribution is then statistically evaluated and shown in a histogram. For this purpose, the 'law of large numbers' is used, according to which an experiment that is often repeated at random but takes place under the same conditions converges to the theoretically most probable value. Even complex relationships can be modeled via Monte Carlo simulation without having to describe them in detail mathematically using formulas. Even in cases where no mathematical solution is known, the Monte Carlo simulation can still approximate the target values.

[2. Definitions

2.1 Probability distribution

The Monte Carlo simulation in antares RiMIS® supports different distribution functions.

The best known distributions are:

- Gaussian normal distribution (continuous, minimum and maximum) → ∞)
- Uniform distribution (continuous, all values within the limits equally probable)
- Log-normal distribution (continuous, maximum 0, minimum → ∞)
- Binomial distribution (discrete values)
- Triangular distribution (continuous, minimum and maximum limited)

In addition, the following distribution functions are supported:

- PERT distribution (continuous, minimum and maximum limited)

In principle, all known probability distributions (such as, e.g., the Weibull distribution, the Gamma distribution, etc.) can be incorporated. Empirical distributions, which result directly from past values, can also be integrated. We are also happy to accept requests from our customers here.

2.2 Correlations

Correlations describe how strongly different risks are statistically coupled with each other. Linear correlations are used for the most part. A linear correlation of +1, for example, between two risks a and b means that if a takes value x, b always takes exactly value y, which results from x multiplied by a positive factor. A correlation of 0 means that nothing can be inferred about the value of b from knowing the value of a. If the correlation is + 0.9, for example, and a takes a relatively high value, then b is also likely to take a relatively high value.

antares RiMIS®, however, not only supports the mapping of simple linear correlations in the classical sense, but instead implements more specific dependency structures by means of so-called copulas. This means that the coupling of risks can be tighter or less tight, depending on the degree of risk. In this way, a broader spectrum of possible dependencies between risks can be represented more precisely than would be possible with the simple assumption of a linear correlation.

2.3 Copula

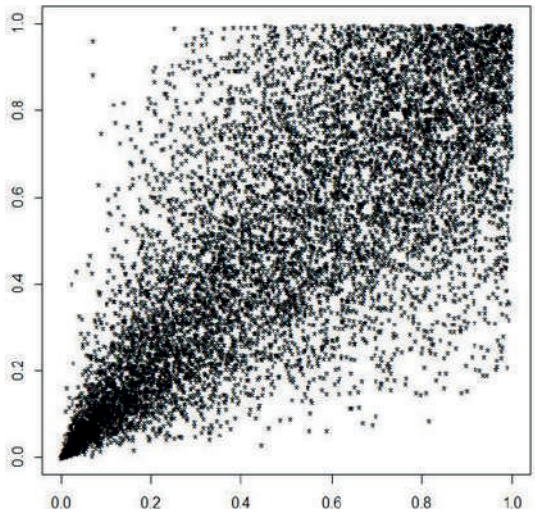
A copula is a joint distribution function of different risks. This distribution has as marginal distributions in each case just the assumed distribution of the individual risks. In this way, copulae are much more flexible than linear correlations, and in particular allow for different degrees of coupling of risk values depending on the level of risk. This is also illustrated by the illustrations of copulae shown below using the example of risk pairs.

In the simulation runs, the distributions of the individual risks are linked according to a defined connection (copula) in such a way that, for example, in the case of a positive correlation in a Clayton copula, a high correlation occurs for low risk values (near best case) and a low correlation occurs for high values (worst case).

antares RiMIS® currently supports three different copulae, which are presented on the following page:

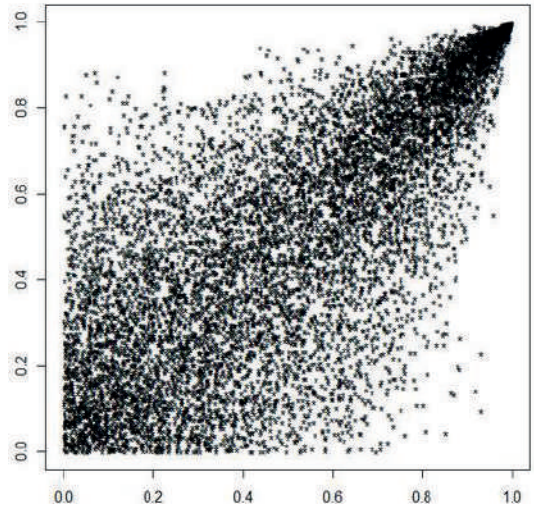
2.3.1 Clayton copula

High correlation at low values (near best case) and low correlation at worst case, distribution is axisymmetric to the origin line.



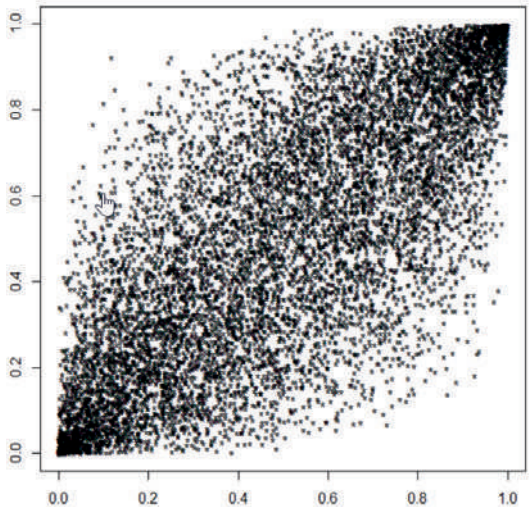
2.3.2 Flipped Clayton copula

High correlation at high values (near worst case) and low correlation at best case, distribution is axisymmetric to the origin line.



2.3.3 Normal copula

High correlation at low and high values, medium in the middle range of values, distribution is axisymmetric to the origin line.

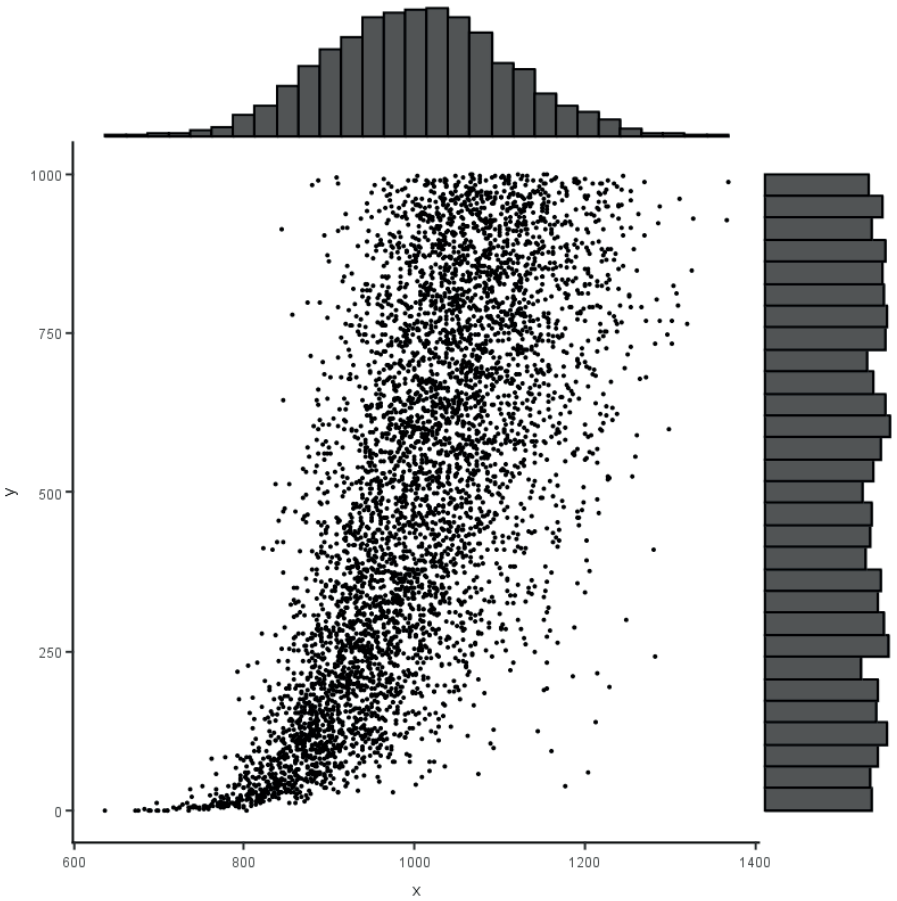


In order to simplify the application, the copula is parameterized via a classical correlation coefficient, i.e., in addition to the selection of the appropriate copula type, the correlation coefficient between two risks, which represents the degree of correlation, must be entered as a parameter. antares RiMIS® facilitates the use of copulas for the user via this kind of parameterization.

The correlation coefficient can be between -1 and +1(-1: complete negative correlation,+1 complete positive correlation; 0: no correlation). The risks generated via simulation then have approximately this linear correlation coefficient, which depends on the selected copula type – but is 'distributed' differently over the range of values.

Example:

2 risks (x and y) with the marginal distributions:
x: Normal distribution (expected value=1000, standard deviation= 100) and
y: Uniform distribution between 0 and 1000
Connected via a Clayton Copula with a correlation coefficient of 0.7



The histograms show the marginal distributions. The points graph does not show the copula, but the combination of copula and distribution of the two risks.

2.4 Simulation scenarios

Simulation scenarios compile various attributes of a simulation into a scenario, e.g., whether a correlation is to be considered by means of copula, which base value is to be used as expected value and which quantiles are to be used as relevant output.

2.5 Drivers

Since version 5.0, driver-based risks are also supported A driver is the underlying value of a risk, e.g., a currency or commodity price. By examining historical values, correlations between drivers can be determined. These correlations are taken into account accordingly in the driver-based simulation, if this is desirable. Driver-based risks are currently assumed to be normally distributed. The standard deviation from a defined period (e.g., 24 months) is determined from the historical values and used together with either the planned value or the last valid driver value as an expected value as parameter for the normal distribution Driver-based risks must currently be recorded as sensitivity risks.

2.6 Sensitivity risks

A sensitivity risk is based on a driver. One driver usually influences several risks. The input parameters for this are deviations, e.g., in EBIT, assuming that the driver swings upward by 10%, for example, and that EBIT changes by a certain value as a result.

Example:

Driver 'Iron ore' increases by 10% (given assumption)
➡ Iron ore' driver falls by 10%, EBIT decreases by EUR 100 thousand (input value)
➡ EBIT steigt um 500 TEUR

The respective basis of the driver modeling can be taken from point 2.5. The dependency between driver and EBIT is assumed to be linear, i.e., no 'jump effects' are currently depicted, e. g. where if a driver increases by 30%, demand will drop precipitously, for example, due to substitution by another raw material or technology.

2.7 Seed/random numbers

In order to obtain a reproducible result with several runs, but still to be able to use random numbers for the simulation, a so-called seed is used, which initially controls the random generator and thus - with identical choice of the seed, repeatedly generates always the same random numbers.

2.8 Simulation run/histogram and aggregation

For each simulation run, random numbers are generated separately for uncorrelated risks. Using the respective distribution function of the risk, a monetary value is determined from this.

If two risks are connected by a copula, a point is generated in a square. Analogously, for n connected risks, a point is generated in n-dimensional space. For example, if the copula has a high correlation value, many points will be close to the diagonal.

At low correlation, the points are more smeared over the entire square. The quantiles of these risks are then obtained via the two coordinates by means of the distribution functions of the risks. These correspond to the simulated value of the respective risk, which is measured with this quantile. Thus, the copula generates the stochastically fitting assignment of the risk characteristics of the two risks. Compared to a simulation with uncoupled risks, this can also be interpreted as a regrouping of the individual values. Different regroupings (~ copulae) lead to different distributions and histograms of the aggregated risk.

After running the simulation, histograms can be used to get an overview of the distribution of aggregated risks.

The frequency of the respective sum within a certain range is then plotted in the histogram. To create a histogram, ranges are formed (the width or the number of bars can be specified, the width of the bars will then be the same in each case). This is done by counting how many simulation values are in the respective range. The following example illustrates how the application of a copula can affect a histogram. In the example, the distribution becomes 'flatter' as a consequence, since more hits are allocated at both ends of the histogram than without the copula. If this simulation were to be carried out 100,000 times, for example, the result would be a graph for the course of the risk exposure for these considered risks (as a rule, the number of risks is significantly larger in reality).

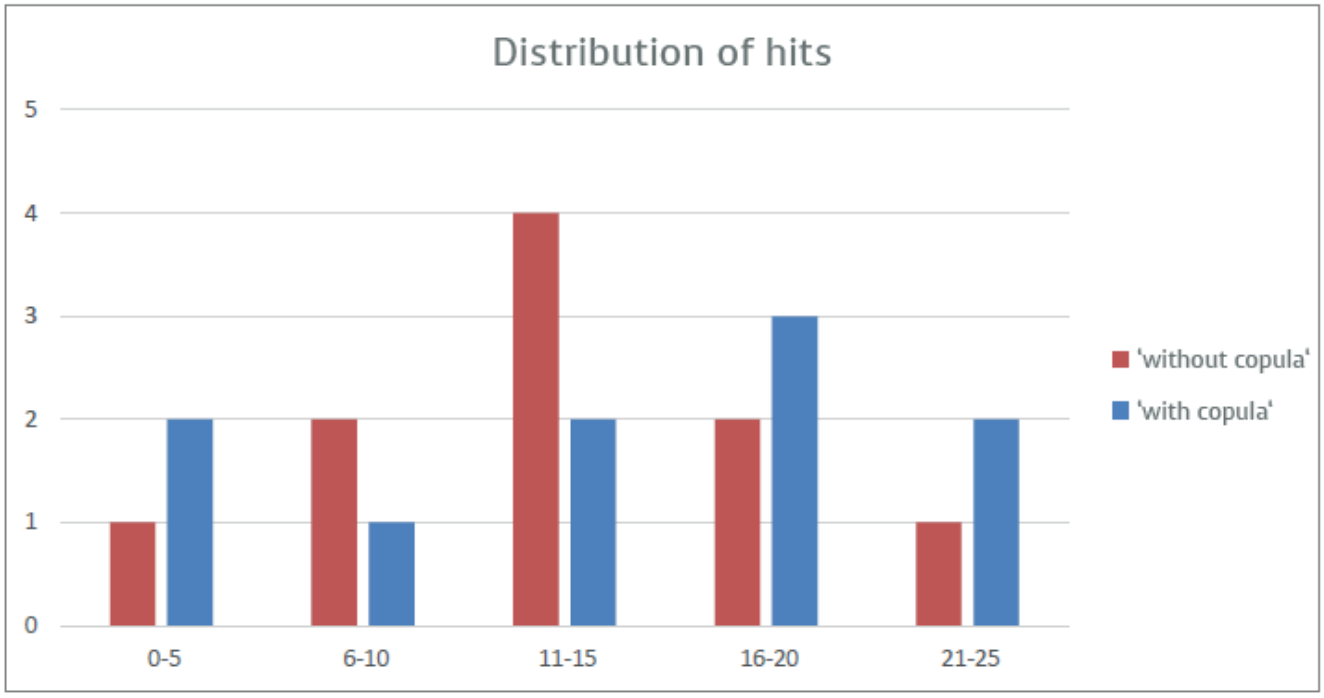
Run	Value R1	Value R2	Value R3	Total R1-R3 without copula	'Rearrange' for example 'normal copula' Value R1	Value R2	Value R3	Total R1-R3 with copula
1	5€	2€	8€	15€	1€	1€	1€	3€
2	8€	4€	9€	21€	1€	1€	2€	4€
3	8€	1€	7€	16€	2€	2€	5€	9€
4	4€	2€	6€	12€	2€	1€	7€	10€
5	5€	2€	8€	15€	4€	1€	6€	11€
6	7€	4€	9€	20€	5€	2€	8€	15€
7	2€	2€	10€	14€	5€	3€	8€	16€
8	1€	1€	5€	7€	8€	2€	9€	19€
9	2€	3€	1€	6€	7€	4€	10€	21€
10	1€	1€	2€	4€	8€	4€	9€	21€

Histogram

Range	Number of runs in the range
0-5 €	1
6-10 €	2
11-15 €	4
16-20 €	2
21-25 €	1

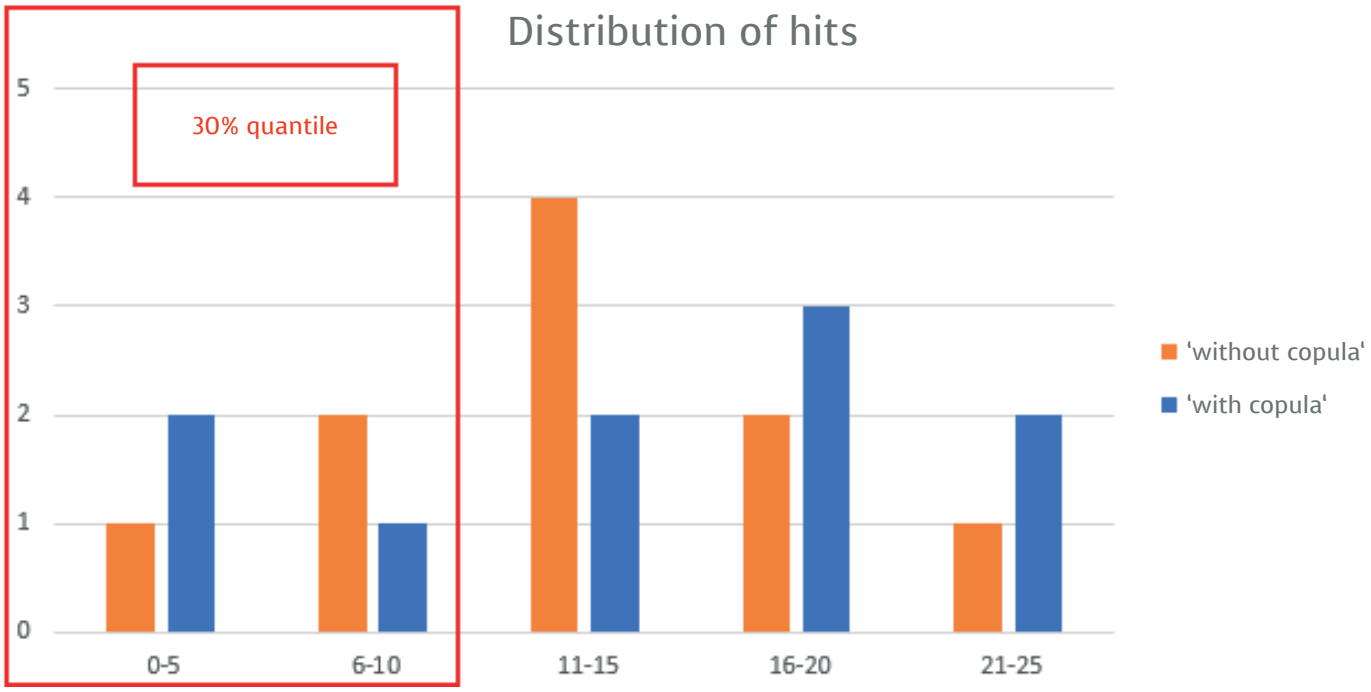
Histogram

Range	Number of runs in the range
0-5 €	2
6-10 €	1
11-15 €	2
16-20 €	3
21-25 €	2



2.9 Quantile of the histogram

A quantile delimits the left area of a histogram in which a certain percentage of the hits are located – e.g., the 30 % quantile in the above example is represented as follows (total number 10. 3 = 30 %):



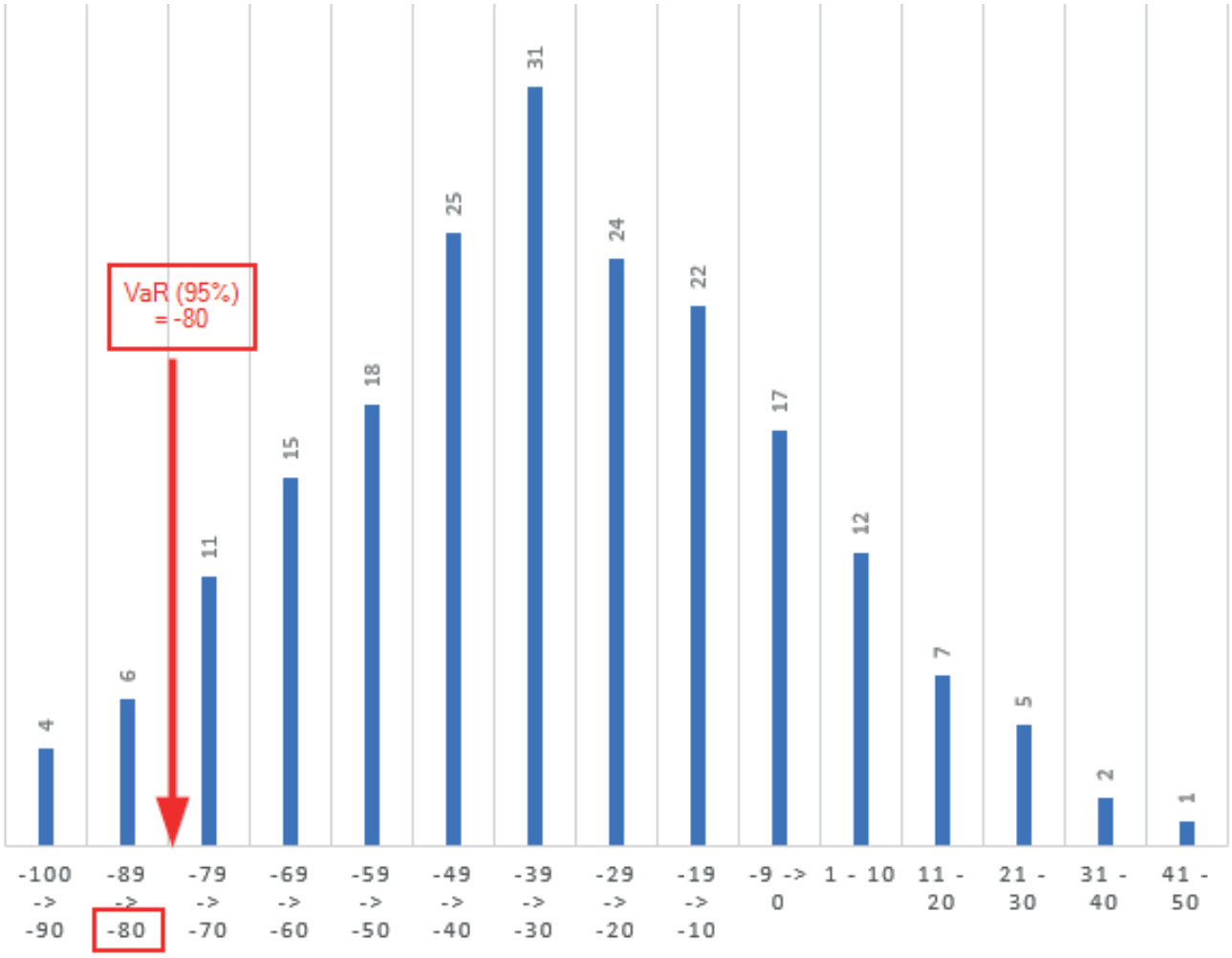
That is, 30% of the hits are in the range of 0-10, or in other words, with a probability of 30% the risk exposure is between 0-10, with a probability of 70% the risk exposure is between 11 and 25 €.

The quantiles are freely selectable and should be defined according to the requirements (e.g., the risk aversion of the company.

2.10 Value at risk

The value at risk denotes the value of a risk that this risk assumes for a specific quantile. In the example below, the VaR (95%) in the simulation is -80, i.e., initially with a frequency of 95% of hits and therefore a probability of 95%, the damage is no higher than -80.

A quantile or a value at risk can thus also be interpreted as follows: The VaR (95%) is exceeded only in 5% of all cases = once in 20 time periods, e.g., once in 20 years. The VaR (99%) is exceeded only once in 100 time periods (years).



The total sum of hits/runs in the example is n=200; 4+6 hits = 10 = 5 % of 200 This determines the limit value. In practice, due to the width of the histogram bars, the value is always only an approximation and afflicted with a certain vagueness – the lower the number of bars and the number of runs, the more blurred the determined value.

2.11 R (programming language)

R is an open source programming language for solving complex statistical problems. R is part of the GNU project and includes, in addition to the runtime environment, a command-line tool that can be used to execute scripts. In R, diverse packages can be installed and also created to solve individual problems. Therefore, it is very easy to add functions other than those already implemented in the standard antares RiMIS® if need be.

[3. Outlook

antares RiMIS® is constantly being further developed. Numerous suggestions for improvement from customers are incorporated into each new version, and the same applies of course to all extension modules.

The following will be further developed in the next versions:

- Automatic simulation of the VaR of all organizational units and comparison of these with the respective earnings contributions of these organizational units, derivation of an EBIT@R.
- Improvement of the input of correlations (concentration on significant risks & opportunities).
- Wizard for the determination of an ideal, risk-specific distribution function by means of a decision tree method.

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